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Research Article

Investigation of Thermo-Electrical Instabilities in a Semiconductor as 2D Dynamical Systems

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Abstract

A semiconducting sample placed in cryogenic media with applied electric field generates low frequency oscillations of electric current and sample temperature and known to be thermo-electrical instabilities. Although observation of current oscillations on oscilloscope is possible, change of sample temperature cannot be detected experimentally. Description of the phenomenon through mathematical equations helps to understand relationship of the two variables as well as their connection to deep trap behavior that are involved in supporting the instability. Mathematical model for thermo-electrical instabilities in an n type semiconductor based on the two deep trap level model with non-degenerate electron statistics has been introduced in order to investigate the unique relationship between the change in time of both electric current flowing through a semiconductor sample and the sample temperature. The 3D dynamical system of nonlinear inhomogeneous ordinary differential equations has been investigated as component 2D dynamical systems (n,T), (n,n_t) and (n_t,T) for local behavior at isolated equilibrium and at points on individual trajectories, where n, n_t and T are free electron concentration at conduction band, electron concentration at deep traps and temperature of a semiconductor sample accordingly. Each of the planar systems is expressed in canonical form and investigated as a Cauchy problem with a set of appropriate initial values. This paper presents investigation results of phase trajectories of the planar systems depending on a single parameter – the temperature of cooling media T_0. Based on obtained calculation results of time sequences of the three variables n, n_t and T, phase differences among these variables have been determined for different values of T_0. It has been established that the change in sample temperature lags behind change in current and this lag increases with T_0. Clearly defined correlations among systems (n,T), (n,n_t) and (n_t,T) are seen, being the result of balance between field aided and thermal ionization mechanisms for charge carrier generation and recombination processes. Thermal and field assisted generation mechanisms compete with one another in achieving steady non equilibrium state in the system depending on temperature of cooling media T_0.

Keywords

Dynamical System, Phase Trajectories, Thermo-Electrical Instabilities, Semiconductor, Deep Traps

1. Introduction

Flow of electric current through a semiconductor placed in cryogenic media at specific conditions oscillates as a result of Joule heating with low frequencies which causes the temperature of the semiconductor sample to oscillate. Studying the relationship of the electric current and temperature variations in time gives an opportunity not only to see how the rela-

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tionship is built, but also how the carrier population of the traps lying deep in the band gap are involved in their own relationship with the variation of sample temperature, and how different generation mechanisms driving the instability coexist in supporting the instability at different temperatures of cooling media T_0. Since it is impossible to register the variations of sample temperature in time experimentally, it is necessary to involve mathematical equations to understand the relationships. The system of ordinary differential equations - the continuity equations, forming the dynamical system describing the process need to take into account generation-recombination processes of charge carriers, based on the, chosen in this case, two trap instability model [1], as well as the equation for distribution of heat along the sample which is well known as the heat conduction equation. Simplifications are unavoidable in such an idealized model system: free electron concentration n and deep trap population n_t are independent of position in the sample, applied electric field value does not change energy spectrum of electrons, a sample is heated uniformly. At the same time temperature dependence of the parameters in the equations like electron mobility, heat capacity, heat conduction, heat transfer coefficient, thermal relaxation time for a sample, band gap width [2-13] as well as mean thermal and drift velocities of electrons, electron lifetime and mean free path between electron collisions in the studied T_0 = (77-197) K. temperature interval for non-degenerate electron statistics must be taken into account. Study of the system (1) was conducted in [14] where time series of variables n, n_t and T have been presented and discussed. Current investigation being an integral part of the conducted study presents results of analysis of obtained phase trajectories of planar systems (n, T), (n, n_t) and (n_t, T) at T_0=(77-197) K.

2. Model Details

The mathematical model for thermo-electrical instabilities in a semiconductor based on two level generation-recombination model by Schoell [1], was introduced in [14] as a 3D (system of three ordinary differential equations) nonlinear dynamical system of nonlinear inhomogeneous ordinary differential equations (1). The system of equations contains two continuity equations for free electrons at conduction band, electron population at deep traps and well known heat conduction equation.

$$\begin{gathered}\frac{dn}{dt} = n^2(-T_1^S - X_1^*) + (N_D - n - n_t)X_{th} - \\ -n(n_t(X_1 - X_1^*) + T_1^S(N_t - N_D) - N_D X_1^*); \\ \frac{dn_t}{dt} = -\left(X_{op} + X_{th}n_t + T^*(N_D - n - n_t) - X_1\right); \\ \frac{dT}{dt} = \left(\frac{k}{c\rho}\frac{d^2T}{dx^2} - \frac{T-T_0}{t_c} + \frac{ne\mu E^2}{c\rho}\right).\end{gathered} \quad (1)$$

The variables and constants in the system are: n=n/N_D, n_t=n_t/N_D, T=T/T_0, t_c, T_0– free electron concentration, electron concentration on traps, sample temperature, thermal relaxation time of a sample and temperature of the cooling media; N_d, E, c, ρ, k, μ – are effective donor concentration, applied electric field (constant), heat capacity of a sample, its density and heat conduction, electron mobility. Heat conduction equation in (1) was set as boundary value problem and expression for distribution of temperature across a sample of size (0.8 x 0.5 x 0.5) cm obtained presented in [15]. The planar systems were set as Cauchy problems with appropriate initial values and real parts of solutions of the initial value problems as time sequences of the variables n, n_t, and T have been presented in [14]. The Results part of this paper, presents phase trajectories of the pointed systems on phase plane for T_0= (77-197) K.

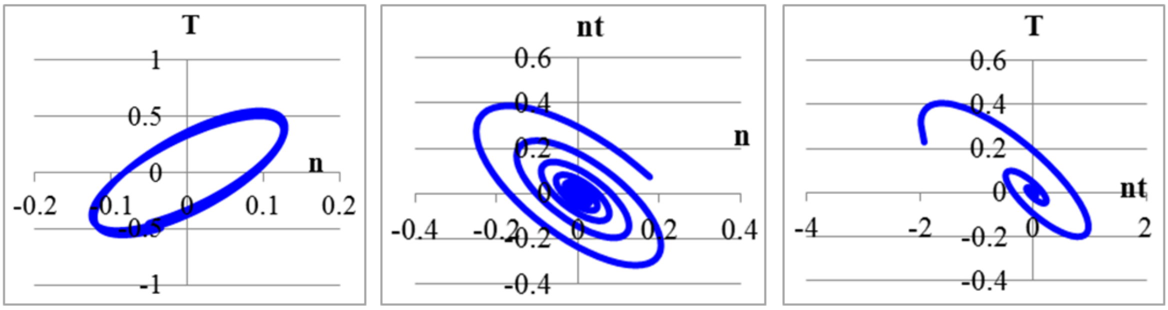
3. Methods

The methods of investigation of the system (1) have been described in detail in [14]. The 3D dynamical system with nonlinear inhomogeneous ordinary differential equations was broken down into planar systems, transformed into “canonical form” by means of non-singular linear transformation [16] and investigated at points on isolated equilibrium and on individual trajectories. Equations of the planar systems were solved with variation of constants method for each of the value of T_0 and real parts of solutions separated and plotted against time to form time sequence of the variables n, n_t, T.

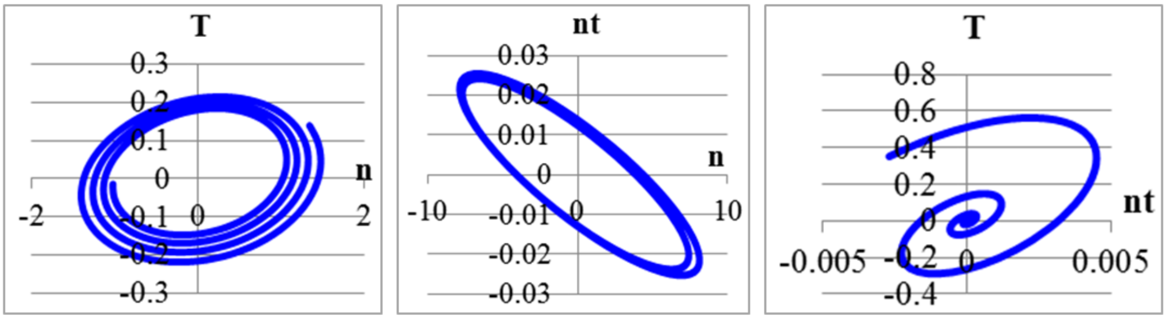
4. Results

Obtained phase trajectories of the planar systems (n, T), (n, n_t) and (n_t, T) are displayed in Figure 1. The variables plotted on the phase portraits are the real parts of solutions of Cauchy problems in [14] for appropriate planar systems. All foci are stable except the one for the system (n_t,T) at T_0=77 K.

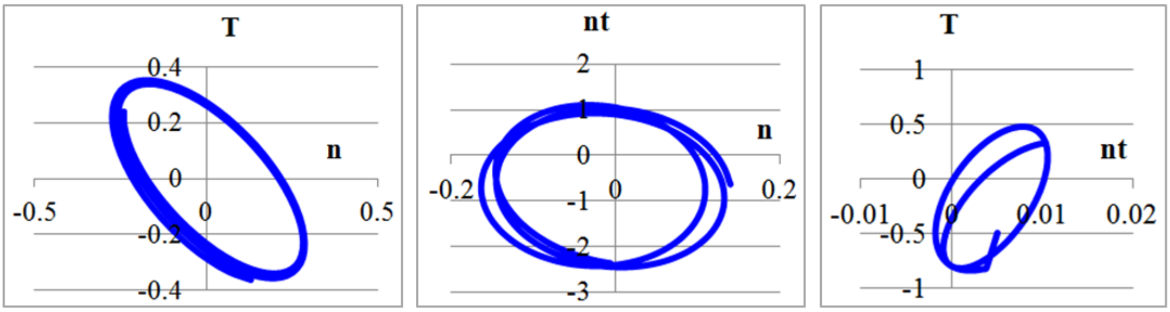
Change in free electron concentration (or current) is followed by change in sample temperature with certain delay, which is reflected in phase difference ϕ between them. With increasing temperatures of the cooling media T_0, ϕ grows for system (n, T), as seen in Table 1. There is a distinct correlation between temperature lagging behind n and n_t, for systems (n, T) and (n_t,T) in the temperature range. While ϕ at T_0=77 K shows a small temperature lag for n, it proves larger for n_t. The opposite is seen at T_0=197 K. For system (n, n_t) values for ϕ are large at endpoints of the temperature range and small at 137 K.



(a)



(b)



(c)

Figure 1. *Phase portraits of systems (n,T); (n,n$_t$); (n$_t$,T), (a) column -* $T_0 = 77\ K$*; (b) column -* $T_0 = 137\ K$*; (c) column -* $T_0 = 197\ K$*. Parameters:* $X_1^0 = 0.01$; $X_1^{S0} = 0.6$; $T_1^S = 3$; $X_1^* = 0.08$; $E = 375\frac{V}{cm}$. *For system (n, T):* $X_1^0 = 0.05$ *at* $T_0 = 197\ K$.

Table 1. *Values of phase difference* ϕ *in radians for the pointed systems.*

T_0=77 K	T_0=137 K	T_0=197 K	
0.754	1.363	2.26	(n, T)
2.19	0.272	1.64	(n, n_t)
1.13	3.32	0.503	(n_t, T)

5. Discussion

The results displayed in Table 1 show clear regularity with which phase difference ϕ changes from one value of T_0 to another for each planar system. As it is seen for system (n, T) at lower temperatures of coolant T_0 ϕ is smaller because the sample cools fast. As T_0 grows so does ϕ, meaning the sample's cooling is taking a longer time, since the sample is heating up. At T_0=77 K for system (n_t, T) field assisted transfer of electrons from deep traps n_t to the conduction band, rather than the thermal one, is the dominant mechanism. The sample's loss of heat is too fast to be able to ionize deep traps, hence a large temperature lag is seen for n_t, which is also responsible for n_t lagging behind n. At T_0=197 K for system (n, T) a large temperature lag for n is coupled with a small one for n_t in system (n_t, T), which means that field aided ionization is no longer the dominant mechanism for carrier generation, but rather thermal ionization. Quick succession of change in n_t

and T results in a small temperature lag for n_t - an indicator of thermal ionization of the traps. Since the limited rate of spontaneous recombination is overwhelmed by thermal generation, and conduction band is not able to accommodate more carriers (because of limited density of states at E_c) being constantly supplied from the traps, a substantial lag of n_t behind n is observed at T_0=197 K. At T_0=137 K, the system goes through a state, in which both the field and thermally assisted ionization of n_t become equal contributors in creating excess charge carriers. The high rates of generation and recombination cause not only a heavy temperature lag behind both n and n_t, but also an almost instantaneous upward transfer of carriers and a downward spontaneous recombination of those, and a very small value of ϕ for (n, n_t).

6. Conclusions

Clearly defined correlations among the planar systems (n, T), (n, n_t) and (n_t, T) show that thermo dynamical equilibrium for each value of the parameter T_0 is achieved through careful balance of thermal and field assisted generation mechanisms. At lower values of cooling media T_0 electric field assisted generation of carriers dominates over thermal generation of the same. It is observed in small value of ϕ=0.754 for system (n, T), which means heat dissipation through sample sides is very effective, and that points to the fact that deep trap population will have to be ionized by means of the most efficient way for T_0=77 K, that being field impact ionization. At T_0=137 K these generation mechanisms balance one another, being equal contributors in carrier generation and transfer of charge carriers between traps and conduction band. That is seen in values of ϕ in Table 1 for each of the planar systems: a) each of the charge carrier generation mechanisms working together effectively in transferring electrons from traps to the conduction band and so supporting the fast recombination of electrons back at the traps showing a very small value of ϕ=0.272 for system (n, n_t); b) relatively large value of ϕ=1.363 for system (n, T) and ϕ=3.32 for (n_t, T) show that change in sample temperature T is heavily “dragging” behind the variations of n and n_t, thus showing the inability of T to keep up with fast transfer of carriers up and down the conduction band and traps due to noticeable difficulty of heat dissipation of sample setting at T_0=137 K.

At high value of T_0=197 K of the coolant, in contrary to the situation with low value of T_0=77 K, thermal ionization dominates over field assisted generation, where the system is trying to reach a new steady non equilibrium state. It is seen clearly in heavy lags (ϕ=2.26) of sample temperature T behind free electron concentration n in Table 1. The value of ϕ=1.64 in system (n, n_t) for T_0 =197 K, as was mentioned above, indicates that fresh supply of electrons from traps being restricted by limited density of states at E_c. For system (n_t, T) with ϕ=0.503 it is a relatively small lag at T_0 =197 K, since the sample has been heated up well and thermal ionization of the deep traps is effective; at the same time T lags far behind n for system (n, T), because cooling of the heated up sample is taking a longer time. In system (n, T) change in temperature lags behind current and the higher T_0 is, the larger this lag. Change in systems (n_t, T) and (n, n_t) is strictly related to that of the system (n, T).

The fact that different charge carrier generation mechanisms can be distinguished from the calculation results of Table 1 is noteworthy. The results give a clear picture of which of the mechanisms dominate the attempt to restore the state of equilibrium, after the system is pushed from it by means of some external force, when recombination and generation rates are out of balance, either in general or in detail.

Abbreviations

2D	System of Two Ordinary Differential Equations
3D	System of Three Ordinary Differential Equations

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Author Contributions

Mukaddas Arzikulova is the sole author. The author read and approved the final manuscript.

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Data Availability Statement

The data is available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest.

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Biography

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